Math 1300 Homework 1

Due July 10, 2023 at the beginning of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. All writing must be done individually.

1. Draw the function $f(x) = e^{x^2}\sin(4x)$ on the interval $[-5, 5]$. (You can use technology.) Recall that a function f is *odd* if $-f(x) = f(-x)$ for all x, and *even* if $f(x) = f(-x)$ for all x. Is f odd? Is f even? In general, what can you say about a function that is both even and odd?

2. Determine which of the functions are invertible with the given domain and codomain, and give an inverse if they are:

   (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^{11} - 22$;

   (b) $f : [0, \pi/2] \to [0, 1]$, $f(x) = \cos(x^5)$;

   (c) $f : [0, \pi] \to [0, 1]$, $f(x) = \sin(x)$;

   (d) $f : [0, \infty) \to (0, 1]$, $f(x) = 1/(1 + x^2)$.

3. A function f has a *root* at $x = a$ if $f(a) = 0$. Find at least one root for each of the following functions:

   (a) $f(x) = x^7 - x^5$;

   (b) $f(x) = \cos(x)$;

   (c) $f(x) = \log(x)$;

   (d) $f(x) = \sin(x) - 1$.

4. Compute the following limits as $x \to 0$. You may use any facts presented in class.

   (a) $\sin(17x)/x$;

   (b) $(x^6 - 1)/(x - 1)$;

   (c) $\sin^2(9x)/x^2$;

   (d) $\sin(1337x)/\sin(420x)$.

5. Compute the following limits as $x \to 1$.

(a) $\frac{(x^2-2x+1)}{(x-1)}$;

(b) $\frac{\sin(x-1)}{x-1} \frac{2^x}{5^x}$;

(c) $\frac{\sin^2(x-1)}{(x^2-2x+1)}$;

(d) $\frac{\sin(\sin(x))}{\sin(x)}$.

6. Consider the function $f : \mathbb{R}^+ \to \mathbb{R}$ given by $f(x) = x^x$. Is there a limit as $x \to 0^+$? You may use technology to guide you to your conclusion. Can you give any explanation for the phenomena you see? (A complete, rigorous answer is not necessary.)

7. Determine which of the following functions have removable discontinuities, including where the discontinuities are and what value can be assigned to make them continuous (recall that $\sin(x)/x$ and $(x^2 - 1)/(x - 1)$ are examples of removable discontinuities). The domain should be considered to be all of $\mathbb{R}$.

(a) $\frac{\sin(x)}{2+\sin^2(x)} + \frac{(x^3-1)}{(x-1)}$

(b) $\sin(\tan(x))$

(c) $\tan(\sin(x)) + \frac{x^4+x^2+5x}{x-3}$

(d) $\frac{\text{sign}(x)}{x}$.

8. Verify that $f(x) = x^8 - 4x^6 - 3x^3 + 8$ has more than one root in $[-2, 2]$. (Hint: IVT.)

9. Explain why the following equations have solutions:

(a) $2023 - \sin(x) = x$;

(b) $\exp(2023x) = x$;

(c) $\sin(x) = x^{2023}$.