Math 1300 Homework 3

Due July 24, 2023 at the beginning of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. All writing must be done individually.

1. (a) Find the derivative of $f(x) = \log(\arctan(x))$ and evaluate it at $x = 1$.
   (b) If $f(x) = \arcsin(2e^x)$, compute $f'(\log(1/5))$.

2. Use the table of values to compute the following quantities.
   (a) $F'(1)$, where $F(x) = f(g(x))$.
   (b) $G'(1)$, where $G(x) = f(x)/g(x)$.
   (c) $H'(1)$, where $H(x) = f(1-x)g(x^2)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	0	24	-36
1	3	4	3	-9
2	9	8	0	0
3	19	12	-3	-94

3. Find the derivative of $f(x) = \operatorname{arccot}(x)$ by differentiating $\cot(\operatorname{arccot}(x)) = x$ and using the chain rule.

4. A 10 foot long ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 feet per second, how fast is the top of the ladder sliding when it is 2 feet above the ground?

5. Estimate the given values using linear approximation, not using any kind of computational aid. (You will need to choose the point where you compute the derivative.)
   (a) $\log(1.1)$;
   (b) $\sqrt{2023}$ (Hint: $2025\ldots$);
   (c) $\sec(0.1)$.

6. A rectangle is inscribed in an isosceles triangle of height 1 meter and base 3 meters. (The rectangle has two vertices on the base of the triangle.) What is the largest possible area of the rectangle?

7. In each case, decide whether a function with the given properties can exist. If yes, sketch a graph of such a function.

   (a) $f(x) < 0$ and $f'(x) > 0$ for all x;

   (b) $f(x) < 0$ and $f'(x) < 0$ for all x;

   (c) $f''(x) > 0$ and $f(x) < 0$ for all x;

   (d) $f(x) > 0$, $f'(x) > 0$, and $f''(x) < 0$ for all $x > 0$.

8. Find the minimum value of $f(x) = x - \frac{4x}{x+1}$ on the interval $[0, 3]$.

9. Let f be a function such that $f'(1) = 0$ and $f''(x) = x^4 - a^2x^2 - 2a - 1$. Which of the followiing conditions guarantees that f has a local minima at $x = 1$?

   (a) $a < -2$

   (b) $-2 < a < 0$

   (c) $0 < a$

   (d) $-2 < a < 2$

   (e) $-2 < a < 1$